$$= \pi dN \operatorname{Cos} \psi \qquad A \operatorname{Sin} \psi n_b \tag{A.2}$$

where ψ is the angle of the helix with the horizontal and A is its area. We have:

$$A/d^2 = \frac{\bar{l}\overline{w}\sqrt{\bar{s}^2 + \pi^2}}{\bar{s}}$$
 (A.3)

and

$$\cos\psi = \frac{\pi}{\sqrt{\bar{s}^2 + \pi^2}} \tag{A.4}$$

$$\sin \psi = \frac{\bar{s}}{\sqrt{\bar{s}^2 + \pi^2}} \tag{A.5}$$

Axial drag flow is therefore:

$$q_d = \frac{\pi^2 d^3 N \overline{l} \overline{w} n_b}{\sqrt{\overline{s}^2 + \pi^2}} \tag{A.6}$$

Multivariable Control of a Wet-Grinding Circuit

The dynamic behavior of a pilot-plant grinding circuit was modelled by relating three output variables that were representative of the conditions within the mill and the classifier to three input variables, namely feed rates of the solids, mill water, and sump water.

A multivariable controller was designed, by the use of Inverse Nyquist Array techniques, for feedback control of the outputs by the inputs, and applied to the plant by a process-control computer with the use of direct digital control.

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SCOPE

Grinding circuits contribute significantly to the capital and operating costs of mineral-processing plants, and much effort has therefore been expended on the understanding of these circuits and optimization of their design and operation. This paper describes a method used in the development of a multivariable control system for a grinding circuit and how the method was successfully applied to a pilot plant.

The control of a grinding circuit to an "operating point" can be regarded as the control of a number of important output variables (such as circulating load, conditions of classifications, and size of product) by means of plant input variables (such as the feed rates of solids and water). This poses a multivariable control problem that can be formulated as: identification of the relation between particular inputs and outputs and the design of a controller to control the outputs by maniputation of the inputs. The theory of linear dynamic systems is very useful in this application, and it is common practice for feedback control to be used to yield good responses that are insensitive to inaccuracies in the linear model of the plant.

Many attempts have been made to partition the multivariable control system by the selection of input-output pairs and the design of a controller for each pair (Gault et al., 1979), since this

enables the application of commercially available single-loop controllers. If the system is highly interactive (in the sense that each input has an appreciable effect on many of the outputs), the restricted structure of such a control scheme could result in poor control due to the interactions between the control loops. In practice, this would often result in one or more of the control loops not being implemented. However, the rapid development of digital hardware has made it economically feasible for a more sophisticated controller to be used that can implement full multivariable control. In general, such a controller computes changes in a number of inputs for each deviation of an output from its setpoint and has special provisions for dealing with interactions.

Inverse Nyquist array (INA) techniques were used for the design of a multivariable controller. The plant and the controller were considered in terms of their transfer-function matrices, and the INA provided a graphical means for the study of the stability of the system and its other characteristics. Fundamental theory relating to the INA can be found in a publication by Rosenbrock (1974), but some novel computational methods were used in this work for the derivation of a dynamic multivariable compensator.

CONCLUSIONS AND SIGNIFICANCE

A pilot grinding circuit has been successfully controlled by a dynamic multivariable controller in a way that allows stable operation at significant setpoints chosen by an operator. It is possible for three important variables relating to conditions within the mill and the classifier to be controlled.

The dynamic behavior of the plant was found to be highly interactive, but the use of INA techniques enabled a controller

Correspondence concerning this paper should be addressed to D. G. Hulbert. 0001-1541-83-6706-0186-\$2.00. © The American Institute of Chemical Engineers, 1983.

to be designed that achieved a suitable degree of decoupling for easy application to the plant.

The methods used in this work can be applied to large-scale industrial grinding circuits and to other plants with interactive

dynamic behavior. The resulting control system can provide a means for optimization of the operation of a plant in terms of variables that are important for practical and economic reasons.

INTRODUCTION

In the preparation of a grinding circuit for control, it is important that all the equipment should be functioning properly and that, wherever possible, undesirable disturbances to the plant should be reduced or eliminated. In a well-run plant, large disturbances to the plant should be infrequent, so the attention to such disturbances should be only a secondary function of the controller, whose main function should be to guide the plant towards, or to keep it at, the optimum operating point.

In the design of an efficient controller for a plant with complex dynamic behavior, it was considered necessary that the procedure of identification, design of the controller by computer-aided techniques, and implementation on the plant by process-control computer should be followed.

CIRCUIT

The pilot plant investigated had an overflow ball mill of 0.91-m diameter, in closed circuit with a hydrocyclone of 0.152-m diameter. The material ground in the circuit was a phoscorite ore that had been crushed to size smaller than 10 mm. The circuit could treat the ore at a rate of up to about 0.2 kg·s⁻¹. The sump was a relatively small stirred tank with a holdup of about 0.02 m³. A diagram of the circuit is given in Figure 1.

On-line measurements of the process were: the mass of the hopper and the speed of the belt for fresh solids; the flow of water to the mill and to the sump; the flow and density of the slurry to the cyclone; the density of the overflow from the cyclone; the level of the slurry in the sump; the electrical power drawn by the mill; and the torque applied in turning the mill. The mill was supported on pressurized-oil bearings with relatively little friction. Actuators that could be driven by computer adjusted the rates at which the solids, the sump water, and the mill water were added, and the speed of the sump pump.

All the instrumentation on the plant was connected to a process-control computer (which had a 32K core of 16-bit words) for direct digital control. Programs were written in an extended Fortran language that included real-time facilities for control. The same computer was equipped with suitable graphic facilities for carrying out interactive data-processing.

PREPARATION OF THE PLANT FOR CONTROL

Before experimental runs could be carried out for identification of the grinding circuit, certain aspects in regard to the improvement of the plant and to the basic procedures for its operation had to be considered. These aspects included the correct sizing of the slurry pipelines, the shielding and routing of cables to reduce electrical noise on the measurements, the prevention of blockages in the chute feeding the mill, the minimization of the size segregation in the feed hopper, and the design and operation of the sump. The instrumentation was carefully calibrated and maintained to give accurate measurements.

Computer programs were written to monitor and control the plant. The monitoring program scanned all the measurements and stored the filtered values in a disc file every five seconds. The control program implemented direct digital control by making adjustments to the actuators that varied the feed rates of solids and water, and the pump speed. The setpoints of the fast-acting control loops for the feed rates of the solids and the water were set to constant values for open-loop tests, or calculated by a subroutine for the implementation of closed-loop multivariable control.

Control of the level of the sump by the pump speed was treated as a single-loop system. In an attempt to maintain a smooth flow of slurry while preventing the sump from overflowing or running dry, the pump speed was set according to an algorithm that used measurements of the level, flow and density of the slurry from the sump and the flow of fresh water to the sump.

When control of the inputs to the plant and of the sump level were satisfactory, the system was ready for study as a multivariable system having three input variables, namely the setpoints for the feed rates of the solids (SF), mill water (MW), and sump water (SW), respectively. These inputs were available for controlling up

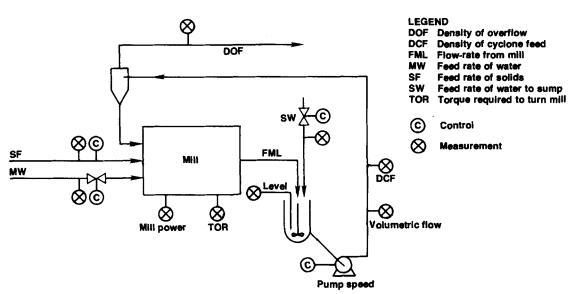


Figure 1. Pilot plant and its instrumentation.

to three observable output variables from the plant that could be selected according to their controllability and importance in regard to the efficiency of the operation of the plant.

IDENTIFICATION

Dynamic tests were performed on the open-loop system during the modelling of the plant. In each test, one of the inputs (SF, MW or SW) was perturbed while the other two were kept constant, and all the measurements on the plant were monitored. The bulk of the experimental runs involved step tests, but some data were obtained from runs with inputs perturbed according to a pseudorandom binary sequence (PRBS). Care was taken to ensure that the steps of the input signals (for SF, MW and SW) were as large as possible while having no adverse effect on the operating conditions of the plant. For example, the cyclone underflow was to be a spray discharge not too far removed from the point at which the underflow assumes the form of a rope, and slurry densities, flowrates, and particle sizes were not to be "abnormal." The step tests alone were actually found to give consistent and adequate results for the derivation of a suitable dynamic model of the plant.

Each open-loop step test entailed a preliminary settling period of half an hour or more with constant inputs, the change of one of the inputs to a different value, and a subsequent period for the plant to respond to the change. All the variables monitored by the computer were logged for processing at a later stage.

The recorded measurements could be used to obtain the responses of many selected output variables to changes in the inputs. A suitable transfer function relating an output to an input could be calculated by the fitting of curves in the time domain (to data from step tests) or by frequency-domain analysis (of data from PRBS tests). The form of the transfer function was selected by inspection of the results of the step tests.

Figure 2 shows the measured and modelled behavior of three output variables—the torque required to return the mill (TOR), the volumetric flow of the slurry through the mill (FML), and the density of the feed to the cyclone (DCF)—in response to step change in MW. TOR and DCF were measured direct, but FML

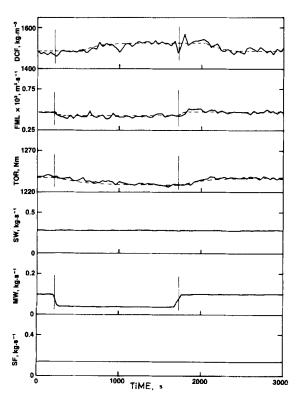


Figure 2. Measured and modelled responses to steps in MW.

was calculated from measurements giving a mass balance round the sump. The responses of FML and DCF are in accordance with the results one might have expected; i.e., they are directly explicable by the dynamics of transport through the mill. The value of TOR is probably closely associated with the holdup of solids in the mill. The response of each output to each input could be described suitably by first-order dynamics, except in the response of TOR to changes in MW, where a dead-time lag was also needed. The responses to changes of SW were all fast, whereas those of SF were all relatively slow.

SELECTION OF OUTPUT VARIABLES FOR CONTROL

The selection of suitable output variables from the plant was based on considerations of the observability, controllability and importance from a metallurgical point of view. With three free input variables, it was to be expected that three outputs could be selected for control. Although the control of three outputs could certainly not guarantee control over all the possible states of the grinding circuit, it was considered feasible for outputs to be selected whose current values and histories in time would be sufficient to imply that conditions within the circuit were within a usefully small subset of all the possible conditions.

The output variables adopted for control were TOR, FML and DCF. TOR and FML were regarded as being descriptive of the conditions within the mill, and DCF as relating to conditions within the cyclone. As was the case with all selections of outputs considered, this selection gave a highly interactive system where each input significantly affected all the output variables.

No on-line measurement of product size was available for use as an output variable. However, the application of a control scheme to an industrial plant would always involve the particle-size distribution of the product. If this is not used directly in the control scheme, it could be used in the selection of setpoints for the controlled outputs, according to the results of correlations calculated from measurements obtained at steady states.

DESIGN OF THE MULTIVARIABLE CONTROLLER

The dynamic model, which was derived for the plant from the results of the open-loop tests and used for the design of a controller, was as follows:

$$G(s) = \begin{bmatrix} \frac{119}{1 + 217s} & \frac{153}{1 + 337s} & \frac{-21}{1 + 10s} \\ \frac{0.000370}{1 + 500s} & \frac{0.000767}{1 + 33s} & \frac{-0.000050}{1 + 10s} \\ \frac{930}{1 + 500s} & \frac{-667e^{-320s}}{1 + 166s} & \frac{-1,033}{1 + 47s} \end{bmatrix}$$

with inputs SF, MW and SW (corresponding to the columns of the matrix) and outputs TOR, FML and DCF (corresponding to the rows). The unit of measurement for each gain is the ratio of the unit of the corresponding output and input. The INA of the model, after preliminary scaling of the inputs and outputs, is shown in Figure 3

The basic use of the *INA* in the design of controllers is described by Rosenbrock (1974). More of the underlying theory is given by Rosenbrock and Storey (1970). The *INA* can provide a graphic representation of the open-loop response of the combined controller and model of the plant. The rows of the *INA* correspond to setpoints specified for the controller, and the columns correspond to the outputs of the plant. If each setpoint affected mainly its corresponding output, with little effect on other outputs, the diagonal elements in the *INA* would be large compared with the off-diagonal elements. Diagonal dominance is required in the design so that a high quality of control can be achieved that takes interactions into account and allows simple on-line tuning when the controller is applied to the plant. The decoupled or partially decoupled loops.

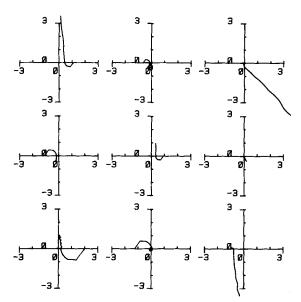


Figure 3. INA of model after preliminary scaling.

represented by the diagonal elements of the *INA*, may be analyzed by conventional single-variable control methods, provided by the Gershgorin bands (Rosenbrock, 1974) are taken into account. Gershgorin bands, plotted on the *INA*, can be regarded as indicating "areas of influence" on the diagonal setpoint-output pairs by the off-diagonal interactions.

An important part of the design was the construction of a precompensator to give an open-loop system that was diagonally dominant. Attempts to derive a suitable compensator by pseudodiagonalization, according to a method of Rosenbrock (1974) were unsuccessful. An interactive computer program was written for the design of a dynamic compensator. Conventional unit matrix operations, as described by Rosenbrock (1974) were carried out by the program, but two new computational strategies were introduced to facilitate the design. These were an adaptation of the methods of Gaussian elimination and the optimum scaling of rows and columns of the *INA* to maximize, or to tend to maximize, diagonal dominance.

The elimination or reduction of an off-diagonal element of the INA is carried out where necessary by a row operation (for a precompensator) involving a rational function, R(s). This row operation is the addition of the product of $R(j\omega)$ and a row k of the INA, to another row i of the INA. If the intention is to reduce an offdiagonal element i,j of the INA, the theoretical operator for complete elimination, $R^*(j\omega)$, is minus the ratio of the elements i,j and k,j of the INA. The theoretically exact row operator, $R^*(j\omega)$, can be plotted together with a guessed or fitted curve, $R(j\omega)$. The designer has control of the structure of R(s), so that he can avoid introducing unwanted qualities such as poles in the right half of the complex plane. A regression routine is used to optimize the parameters of R(s) so that the size of the resulting off-diagonal element of the INA can be minimized. In this way, by a series of row operations, some or all of the off-diagonal elements are reduced by application of a method that is analogous to Gaussian elimination as applied to matrices of real numbers instead of matrices of

The optimum scaling of the rows and columns of the INA by scalors is a useful technique that does not add to the functional complexity of the resulting compensator. It results in the reduction of relatively large off-diagonal elements at the expense of increasing smaller ones. This operation may sometimes actually bring about the condition of diagonal dominance. It is best applied at the beginning of the design when the system has arbitrary units of measurement determined by the inputs and outputs of the plant, and after the reduction of off-diagonal elements by the quasi-Gaussian elimination.

The design of a dynamic compensator for the model, G(s), of

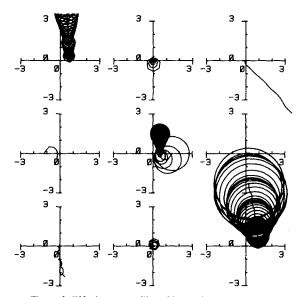


Figure 4. INA of system with multivariable compensator.

TABLE 1. NONZERO ELEMENTS OF MULTIVARIABLE COMPENSATOR

Matrix Element	C	au(s)
1,1	0.0756	0
1,2	-0.4771	ő
•	-0.5969	238
2,2	0.6603	0
3,1	0.0661	496
3,2	1.837	461
	0.2141 + i0.0642	59.3 + i39.7
	0.2141 - j0.0642	59.3 - i39.7
3,3	-0.00182	0

the grinding circuit was successful. The resulting INA, with Gershgorin circles (Rosenbrock, 1974), is shown in Figure 4. The dead-time lag in the 3,2 element of G(s) presented no problems in the design, and the final diagonal terms had no right-half plane zeros or poles. Table 1 gives the nonzero elements of the compensator, expressed in terms of coefficients C and time constants τ of their partial fractions of form $C/(\tau s + 1)$.

The curves and Gershgorin bands of the diagonal elements in Figure 4 do not cross the negative real axis; this implies that, if their gains are large enough, the outputs could theoretically be controlled arbitrarily well, if there were no restrictions on the plant inputs. The large Gershgorin circles could be tolerated because of the high gains possible. After some simulations were carried out with the controller and the model, G(s), the controller was applied to the plant, and the final tuning of the three partially decoupled control loops was carried out.

IMPLEMENTATION OF THE CONTROLLER ON THE PLANT

The multivariable conpensator was implemented by a subroutine in the control program. Another subroutine provided for proportional-integral-derivative (PID) control elements for each of the partially decoupled loops. Specific experimental tests were carried out to yield suitable parameters for the PID elements. The multivariable compensator was fixed, but the PID parameters were variable on-line, and these could be adjusted to achieve the best results, according to the effects of noise, input limits and nonideal responses of actuators. It was found that the multivariable compensator had indeed decoupled the system fairly well, so that tuning of the three PID controllers could be done independently and relatively easily. The parameters finally adopted for the PID control loops are given in Table 2.

TABLE 2.	PARAMETERS OF PID CONTROL	ELEMENTS
ontrol	Integral	Deriva

Control Loop	Gain	Integral Times (s)	Derivative Time (s)
TOR	0.1	100	10
FML	0.3	100	0
DCF	0.5	50	10

The performance of the controller was tested by experimental runs in which setpoints for the output variables were given step changes. Figure 5 gives the results of one such run, which illustrates a number of points in the operation of the circuit under computer control. The setpoints for the outputs are indicated by dotted lines. The figure shows that the controller performed well when it was activated soon after startup of the circuit. Initially, all three outputs were significantly different from the setpoints, but were soon driven to the setpoints.

During the run, the first change in the setpoint for TOR was followed quickly and satisfactorily. After that, due to an error, the setpoint for TOR had been specified as 12,400 Nm instead of 1,240 Nm for a short period, causing SF to be limited at its maximum. An abnormally low setpoint was then given, and the response of TOR was relatively slow, because the controller was restricted by the lower limit of SF and the upper limit of SW. The operator made two further changes in the setpoint for TOR, to which TOR responded quickly, the first of which produced a slight overshoot as a result of the long period of saturation at the lower limit of SF.

Towards the end of the run illustrated in Figure 5, a change was made in the setpoint for *FML*. This was a noisy variable, but its response to the change in setpoint was good.

A major restriction of the response of *TOR* was the slowness and the dead band of the actuator of the solids feed. For example, when a large increase in torque was required and *SF* went to its maximum limit, a large overshoot could occur if *SF* could not be reduced quickly enough as *TOR* approached the new setpoint. Derivative action was found to counter this nonlinear effect.

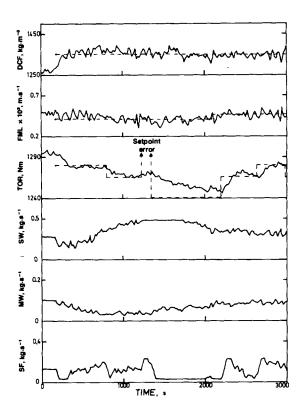


Figure 5. Results of a run with multivariable control.

Of the three outputs, the *DCF* was probably the best controlled. The very short-term noise could not be eliminated, because there was a dead-time lag of about three seconds due to the length of piping from the sump to the density gauge.

OPERATING LIMITS OF THE PLANT

The control of the plant facilitated an investigation of when and how the plant had reached its operating limits. This type of information about a plant is very useful in practice, because the optimum operating point is very often at a limit, and the limit might be identified as something that could be extended cheaply, for example, by the modification of a cyclone. The pilot plant used in the investigation was run intermittently and without any realistic basis for an economic study. However, on an industrial plant, the economic aspects are very important.

The torque could be controlled over a relatively wide range, determined mainly by the limits of SF. When the upper limit of SF was raised sufficiently, the upper limit to TOR was imposed by the condition where the mill became choked and the efficiency of grinding dropped.

The range over which FML could be controlled was restricted by the range of the input variables. Any one of the three inputs could cause a restriction, depending on the operating levels of TOR and DCF. The mill itself did not restrict the range. The choice of a setpoint for FML relates to the desired circulating load and would normally be decided by economic factors.

DCF could be controlled over a wide range. The lower end of the range allowed slurry densities that were too low for the satisfactory operation of the mill, especially in terms of wear of the linings and the grinding media. The upper limit of *DCF* was determined by pumping limitations.

CONCLUSIONS

The grinding circuit investigated can be controlled to have torque, flow through the mill, and density of the cyclone feed following setpoints within practical limits. With such control, investigation of the optimum operating conditions for a grinding circuit is greatly simplified, since the circuit can be made to operate in any practical region of the operator's choice. On an industrial plant, the main object would be the maximization of financial profit, although a number of more direct criteria could be considered. These might include maximization of the throughput at a specific average size of product, minimization of the size of the product at a given feed rate, limiting the production of very fine material and the avoidance of any particular practical problems of operation.

A grinding circuit has an interactive dynamic behavior. Multiple single-loop control systems generally fail to deal adequately with interference between the control loops, and the tuning of the loops is difficult or impossible, with the result that one or more of the loops may be dropped from the control scheme. However, the multivariable methods used in this work were found to overcome the interactions and produce a successful control scheme.

Optimum control of a complex grinding circuit presents difficulties, but these can be overcome by the derivation of a control scheme that brings the dynamics of the circuit under control, followed by a study for determination of the best operating conditions to be maintained by the controller. A controller designed by *INA* techniques gave satisfactory results on the pilot plant, notwithstanding the fact that the noise levels on the plant were relatively high, and the dynamic response of the plant was represented by a simple model. The final on-line tuning of the controller was relatively easy. The choice of optimum setpoints could best be made in a study of the controlled plant, special attention being paid to composite samples of the product from the circuit, and, if relevant, the behavior of downstream processes.

NOTATION

DCF = density of the cyclone feed (kg·m⁻³) DOF = density of the overflow $(kg \cdot m^{-3})$ FML= flowrate from the mill $(m^3 \cdot s^{-1})$ G(s)= matrix transfer function for plant

= inverse Nyquist array INA = number $\sqrt{-1}$

MW= feed rate of water to the mill (kg·s⁻¹) PID = proportional, integral and derivative PRBS = pseudorandom binary sequence R(s)= rational function used in row operation

 $R^*(j\omega)$ = theoretically exact row operator

= Laplace variable

SF = feed rate of solids (kg·s⁻¹)

SW= feed rate of water to the sump $(kg \cdot s^{-1})$ TOR = torque required to turn the mill (Nm)

= frequency $(rad \cdot s^{-1})$

ACKNOWLEDGMENT

This work, done at the University of Natal, Durban, Republic of South Africa, is published by permission of the Council for Mineral Technology.

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Disruption Mechanism of Aggregate Aerosol Particles through an Orifice

Based on the drag force of two spheres in contact and the probability density of size ratios of particles which constitute an aggregate particle, the population balance equation describing the change of the particle-size distribution due to the disruption of aggregate particles is derived and the numerical solutions of this equation are obtained.

Experiments were carried out with fly ash particles dispersed in air stream through an orifice at various flow rates. The measured size distributions can be represented by the numerical solutions of the population balance equation.

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SCOPE

Little attention has been focused on the disruption mechanism of agglomerated solid particles in fluid stream (especially in air stream), even when both mechanisms of liquid and solid particle agglomeration and liquid droplet splitting in immiscible liquids have been studied theoretically and experimentally by many authors (for example, Hinze, 1955; Saffman, 1956; Ramabhadran et al. 1976). The reason for this may be the complicated feature of the solid aggregate which consists of particles in contact in which the original particles can be recognized yet. However, reliable estimates of the size distribution of dispersed solid particles in aerosol streams are required in many chemical engineering, for example, in the design of various dust collectors and of atomized fuel injection systems. The principal objective of the present study is to reveal the disruption mechanism of aggregate particles by predicting theoretically the changes of particle-size distribution through an orifice and measuring them

Patterson and Kamal (1974) and Kamal and Patterson (1974) studied the shear deagglomeration process of solid agglomerates suspended in viscous liquid. They proposed a model to predict the equilibrium and transient particle-size distributions as a function of shear stress, the initial particle-size distribution and the agglomerate strength distribution. They, however, only

treated the deagglomeration process of the agglomerate which consisted of uniform-size particles. Real agglomerate consists of various size particles.

With a view to predicting the manner of break-up of aggregate under fluid forces, Bagster and Tomi (1974) examined the stresses on planes within a sphere in simple flow fields. They assumed that an aggregate is a sphere. The assumption is acceptable for the break-up of droplet, but is not acceptable for the disruption of solid aggregate. Kousaka et al. (1979) proposed the possible disruption mechanisms of aggregate particles in air stream and indicated macroscopically that impaction of particles on some obstacles and acceleration (or deceleration) of aggregates in air stream are effective in aggregate disrup-

In the present study we derive the equation describing the change of the particle-size distribution due to the disruption of aggregate particle, which consists of various size particles, by considering the relative velocity of aggregates to fluid, the drag force of two contact particles and the probability density of size ratios of particles which constitute an aggregate particle, and give the numerical solutions of this equation for various conditions. The changes of the particle-size distributions due to the disruption by the acceleration of aggregate particles through an orifice are measured. The calculated and the measured values are compared, and the disruption mechanism is discussed.